MATHEMATICAL KNOWLEDGE IN A MATHEMATICS TEACHING EPISODE

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This paper addresses issues relevant to an analysis of mathematical knowledge in a mathematics teaching episode. Moving from a concern with what it terms the 'teaching paradox', the paper seeks to analyse an episode taken from a year 11 mathematics classroom. In this analysis three main concepts are used: knowledge base, knowledge artefact, and functional knowledge claim. The main finding of the analysis is that mathematical knowledge stands in a peripheral relation to mathematics teaching.

BACKGROUND

In their book *Situated learning: legitimate peripheral participation* (1991), Lave and Wenger make the observation that normal classroom practice revolves around a teaching curriculum rather than a learning curriculum. What this amounts to for mathematics education is that teachers do not teach 'mathematics' so much as teach 'mathematics teaching'. If true, this observation would seem to have paradoxical implications, such as, the more teachers teach mathematics, the less mathematics is taught, and so on. These kinds of points have, of course, also been made by others in the mathematics education literature, most notably Brousseau (1986) and Steinbring (1989), where they have been linked with studies investigating classroom interactions from micro-sociological perspectives. I will refer to this paradoxical relationship between 'mathematics teaching' and 'mathematics' as the 'teaching paradox' and will use this as a starting point for an investigation into mathematical knowledge in a mathematics teaching episode.

In seeking to better understand this paradox, I have turned initially to Shulman's now classic typology of teacher knowledge (1986, 1987). In this, teachers' knowledge is posited to consist of content knowledge, curriculum knowledge (general knowledge of how to teach), pedagogic content knowledge (the knowledge of how to teach particular topic areas of content knowledge), and other kinds of knowledge relating to the needs of students and characteristics of the context of teaching. Of these, possibly content knowledge and pedagogic content knowledge are of most direct interest to my study; they can be taken to roughly equate with 'mathematics' and 'mathematics teaching', respectively. For, holding both the disciplinary needs of mathematics and the learning needs of students clearly in view, expert teachers work through the curriculum topic by topic, making and implementing along the way reasoned judgements based on their experience, knowledge and belief of what good mathematics teaching amounts at each stage of progress.

Whilst Shulman's contribution has been enormously helpful in political debates surrounding the status of teacher knowledge and models for teacher education, when viewed from the perspective of the teaching paradox, however, it presents as strangely inadequate. For if the goal of good teaching is to advance mathematics learning, then according to the teaching paradox, the place of pedagogic content knowledge (mathematics teaching) surely ought to be less prominent. Instead, however, Shulman places this body of knowledge at the centre of teachers work and insists that expertise in this body of knowledge is at the heart of teaching expertise. In trying to explain this state of affairs, one option to be considered might be that the language used has become entangled, that precision has been lost, and that this have given rise to meanings contrary to intentions. Another option could be that teaching simply does involve paradoxical relationships, and that learning occurs precisely because of them, not despite them. Whether one or both or some other alternative is the case, I would suggest that developments in knowledge of teaching and learning are bound up, in part at least, with these matters.

In this paper, I want to explore an aspect of the teaching paradox. Using data obtained by video-taping a Year 11 mathematics class in operation, I will seek to demonstrate how Shulman's pedagogic content knowledge can overshadow mathematical knowledge in an actual teaching situation. My analysis will lead to a new model of relationships among the knowledge bases of the teacher and learner. The paper will conclude with a statement of implications for teaching and learning and further research.

Theoretical Framework

Consistent with cultural historical perspectives on learning initiated by Vygotsky (1978) and others, my study attempts to build an epistemological analysis of interactions observed, by analysing task performance. By a task, I mean, roughly, an activity with a definite purpose or object in view. Whilst analyses of this kind are possible using a variety of alternative perspectives (eg reflective practice (Schön, 1987); social constructivisim (Cobb, 1994); discourse theory (Walkerdine, 1988); sociology (Dowling, 1998)), the approach I am attempting is oriented towards synthesising activity theory concepts to cognition with matters arising within the philosophy of knowledge (see Fenstermacher, 1994; Kessels & Karthagen, 1996; Thomas, 1997). For such an attempt, I will make use of the following conceptual tools: knowledge base, knowledge artefact, and functional knowledge. Purposes these are put include: identifying, analysing and describing instances of knowledge; clarifying the language used about teaching and learning; exploring assumptions concerning these concepts. In constructing the 'tool-kit' (see Well (1996)), I have sought ideas which are shaped by the conceptual environment in which the problems are posed, for instance, (i) the concept of the social construction of knowledge, (ii) the activity dependent characteristics of knowledge, (iii) the generation of knowledge identity (that which enables it to be asserted that one piece of knowledge is an instance of another or is of the same kind as another), and (iv) the generation of knowledge *difference* (that which, in the language of Gaston Bachelard, induces epistemological rupture and projects the reconstitution of knowledge). I have set these out briefly in the following list.

Knowledge base: Following Jong & Ferguson-Hessler (1993), this term is used to describe a body of knowledge types (situational, declarative, procedural, strategic) and knowledge qualities (surface/deep, isolated/structured, automated, modality, general/domain specific) salient to task performance. Whatever knowledge, described by type and quality, is used in performing a task belongs to the knowledge base of that task. A teacher's knowledge base consists of all the required by the teacher to perform the tasks of teaching. Likewise a mathematics knowledge base, consists of all the mathematical knowledge required in order to perform mathematical tasks.

Knowledge artefacts: I have borrowed the term 'artefact', a "product of human art and workmanship" (*Shorter Oxford Dictionary*), from Vygotsky (1978), Leont'ev (1981) and Lave & Wenger (1991) who refer to artefacts as physical, linguistic and symbolic entities; and used it to coin the new term 'knowledge artefact'. The purpose of this term is to emphasise that an object is meaningless and literally useless unless knowledge, in the form of a context, is brought to it. For this reason, the same object could be any number of different knowledge artefacts, depending on the varieties of contexts brought (for example, think how many purposes we have for a blade, each having a place within their own sphere) and, conversely, different objects could be the same knowledge artefact (think of 'text' - it could be hard copy or on monitor, etc). These examples suggest that it can be important to be able to distinguish between an entity's object status and the status an entity has in advancing a purpose or task. It is also assumed that the following can be said of 'knowledge artefacts': (i) they are human products, (ii) they produced within the context of a set of socially/culturally constructed rules, (iii) they hold different values depending on custom, and (iv) they evolve over time, and therefore have a history represented as traces within the tasks they perform used.

Functional knowledge claim: Getting a task done not only requires the necessary knowledge artefacts and requisite knowledge base - it also requires that these actually be implemented. Now, on reflection, 'implementing' involves making a claim in at least two respects. In the first, to implement a course of action is to quit speculating about how things might be, and to declare how, for you, things are placed at the time. In this sense, getting a task done simultaneously involves making a claim. A second way in which 'implementing' a course of action involves making a claim, is as follows. To do something for a special purpose requires both that a proposal for action be formed (that is, that there be an object in view, tools to hand, and the means whereby the task can be performed and managed ready), and that action really be engaged. Because we actually never really know what our action will lead to, our expectation, commitment, values, and beliefs, and the cognitions surrounding them, are engaged, and these help us make selections and come to a view about the direction our action is taking us. This argument suggests that implementing pertinent knowledge involves making, entertaining and testing commitment, having and creating expectations, and so on. It also suggests that claims which achieve the purpose for the sake of which they were originally made, have the strongest grounds for being declared valid. Other validity grounds are also possible - for instance, where the grounds for expectations and commitment associated with an action are well argued or praiseworthy, etc. In order to facilitate discussion of these collateral events in the life of task performance, I have coined the term 'functional knowledge claim'.

METHODOLOGICAL ISSUES

As indicated previously, data analysed in this study was obtained by video-taping a Year 11 mathematics lesson in progress, devoted to the topic of demonstrating that the argument of the product of complex numbers is the sum of arguments of the numbers involved. My method of analysis was as set out in the following. Firstly, I analysed the content of the transcript and partitioned the material into sections and subsections according to the subject matter involved. These will be reported in detail in a subsequent publication. Secondly, one subsections drawn from this analysis was chosen for analysis in this paper. Thirdly, each of these segments were analysed using the tools referred above. In particular, within respect to each transcript segment, knowledge bases and knowledge artefacts were identified. These steps involved making significant assumptions about the validity of inferences made. A full discussion of these, beyond the scope of the present work, is needed. Fourthly, data were analysed using the conceptual tools previously discussed, and this lead to the development of a new model of teaching and learning interactions. Analyses reported below represent work in relation to the third and fourth stages indicated above.

ANALYSIS

A useful start to the data analysis will be to note that for any two complex numbers, the argument of their product is equal to the sum of their arguments. In the terms of my theoretical perspective, this mathematical fact is an artefact within a mathematics knowledge base. Figure 1, provides a demonstrating of this artefactual status.

Figure 1
Mathematical Demonstration that the Argument of the Product of Complex Numbers is the Sum of their Arguments.

Given $z_1 = r_1 \operatorname{cis}\theta_1$ and $z_2 = r_2 \operatorname{cis}\theta_2$,

$$
\begin{array}{llr}
\arg(z_1 z_2) & = \arg(r_1 \operatorname{cis}\theta_1\, r_2 \operatorname{cis}\theta_2) & 1.1 \\
 & = \arg[r_1 r_2 (\cos\theta_1 + i\sin\theta_1)(\cos\theta_2 + i\sin\theta_2)], \text{ by definition} & 1.2 \\
 & = \arg\{r_1 r_2 [(\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2) + i(\sin\theta_1\cos\theta_2 + \cos\theta_1\sin\theta_2)]\} & 1.3 \\
 & = \arg\{r_1 r_2 [(\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)]\}, \text{ using the relevant trignometic identities} & 1.4 \\
 & = \arg[r_1 r_2 \operatorname{cis}(\theta_1+\theta_2)] & 1.5 \\
 & = \theta_1 + \theta_2 & 1.6 \\
 & = \arg z_1 + \arg z_2 & 1.7
\end{array}
$$

Thus, $\arg(z_1 z_2) = \arg z_1 + \arg z_2$, as required.

In contrast to this, Figure 2 presents the teacher's blackboard script for that part of the lesson relating to the topic of Figure 1.

Figure 2
Transcript of Teacher's Blackboard Script

$$
\begin{array}{rlr}
 & \arg(z_1 z_2) & 2.1 \\
z_1 & = r_1 cis\theta_1 & 2.2 \\
z_2 & = r_2 cis\theta_2 & 2.3 \\
z_1 z_2 & = r_1 cis\theta_1 \times r_2 cis\theta_2 & 2.4 \\
 & = r_1 r_2 cis\theta_1 cis\theta_2 & 2.5 \\
 & = r_1 r_2(\cos\theta_1 + i\sin\theta_1)(\cos\theta_2 + i\sin\theta_2) & 2.6 \\
 & = r_1 r_2[(\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2) + i(\sin\theta_1\cos\theta_2 + \cos\theta_1\sin\theta_2)] & 2.7 \\
 & = r_1 r_2[(\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)] & 2.8 \\
 & = r_1 r_2 cis(\theta_1+\theta_2) & 2.9 \\
 & & \\
\arg(z_1 z_2) & = \theta_1 + \theta_2 & 2.10 \\
 & = \arg z_1 + \arg z_2 & 2.11
\end{array}
$$

The transcript following refers to Figure 2, and provides a record of interactions among teacher and students.

T: Excellent. Well done. Well, one person has made the next jump. Good. What do you think is the next stage? *[teacher talking to student]* That one plus that minus that one there. You put them together. Now anyone else get the next step? We've got three people so far. Peter? Hmmm *[continues speaking to individuals]* OK Let's just make the next quantum step. Look at the first relationship. What is it? Who sees what it is, there's quite a few people I can see who've written things down. Steve, what is it, this first thing up here? *[points to 2.7, Figure 2]* It's written on the board, hint hint hint.

S: *[student inaudible]*

T: *[writing 2.8]* Instead of $\theta_1 + \theta_2$ what is it? $\cos(\theta_1 + \theta_2)$. And what's the other thing there? Now most people got to there *[2.7]*. Remember when you're dealing with complex numbers you need to put things together. So you need to put your real parts together and your imaginary parts together. I think that's where a lot of people couldn't see this next jump. OK, now, you got there. Pens down, and let's just think about what this means. You've got $z_1 z_2$, is equal to that. *[referring to 2.7]*. Right, any worries? Right, this *[2.8]* is in trig form. What's the first bit? *[circles $r_1 r_2$]*

S: Modulus

T: Modulus. So in other words we've got two complex numbers we multiply together, and we know that the modulus will be multiplied together, the two moduluses. Alright, what about this thing here? Let's just rewrite that in the other form, we can write that as $\operatorname{cis}(\theta_1 + \theta_2)$. Alright. Now remember what our original, what I originally asked you. I said predict what $\arg z_1 z_2$ *[writing 2.10]* is going to be. Now remember your arg is an angle, and really what we come up with, well perhaps you can tell me, What have we got there? The argument of the product is equal to what?

S: The sum of the angles.

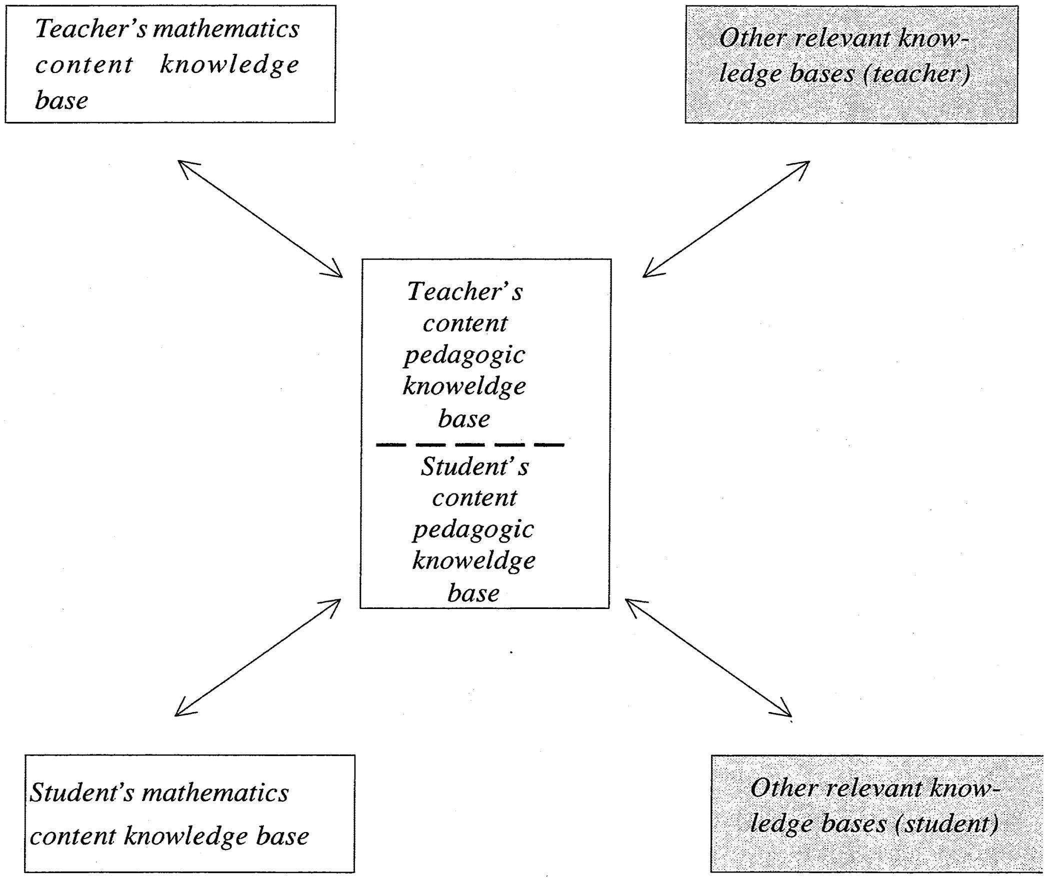
T: The sum of the angles. The arg of this thing here *[2.9]* is $\theta_1 + \theta_2$. OK. So we've got the arg $z_1 z_2 = \theta_1 + \theta_2$. *[2.10]* Right? Now, when we've got $\theta_1 + \theta_2$ what's θ_1 equal to? Come back up to here *[points to 2.2]*, what's θ_1 equal to? It's the arg of z_1 *[writes 2.11]*. And what's θ_2? argz_2. So was our original hypothesis correct?

S: No... *[inaudible]*

T: No, because we predicted - you predicted or someone predicted - who do we blame? Steve? Well he was the only one willing to have a go, predicted that the arg of the product was going to be the product of the arg, which is the normal thing we've had with practically everything we've ever done. But notice here that the argument of two complex numbers is equal to the sum of the individual arguments. OK, so when you look at that it's not what you'd obviously think *[teacher rubbing hands]*. It's going to be something that's slightly different *[teacher looking at students; pause]*. More than slightly different *[teacher walking back to the board]*, dramatically different. We're going from the arg of a product is the sum of the individual parts. Now let's just sum up that before we do anything more with it. So pens down and let's just think for a minute, rather than copying it *[?]* dramatically, because remember what we're after is trying to get some idea of how we go about proving these things. And looking at it. Now, we did a couple of different things today if you look at it. The first thing, what we did was, we developed a relationship. Now the reason we developed that relationship was that we had to use it. I mean, we could have got to there, I could have said to you, oh yeah great, that thing there *[points to 2.7]* is $\cos(\theta_1 + \theta_2)$, but you wouldn't have believed me would you? No you don't believe me in anything do you. OK, so I could have told you that, but the first part of the lesson what we did was we developed this relationship. That when we've got a sum of an angle and we take the cos of that or the sin of that and even we can go ahead and work out the tan of those things, then we can relate it back to the cos and sin of the individual angles. Alright? The second thing that we did was we said righto, let's go now to our arg$z_1 z_2$ *[2.10]*, and what we did, notice we started off with $z_1 z_2$ *[points to]* and we used our trigonometric form and we've gone through and developed something like that. We used our expansion idea *[refers to 2.7]*, we've come back to $r_1 r_2$, and put it all together, and we've gone from taking the separate parts and we've put it into $\theta_1 + \theta_2$ *[points to 2.8]*. So we've ended up with this relationship here *[draws blocks around 2.10 and 2.11]*.

As a comparison with Figure 1 will show, many knowledge artefacts contained in these episodes correspond with or are consistent with the mathematics content knowledge artefact exhibited in Figure 1. This finding is, of course, expected. However, the transcript reveals other kinds of knowledge artefacts. For example, in lines 10-12, the teacher says "Remember when you're dealing with complex numbers you need to put things together. So you need to put your real parts together and your imaginary parts together." Although this statement appears to address the management of complex numbers, it does so *as if* complex numbers were concrete and the management issue concerning them revolved around a sorting operation. As neither of these is the case, it is concluded that this statement is not a mathematics knowledge artefact. However, the statement is certainly relevant to complex numbers in the sense that given the right code it can be translated into a functional knowledge claim capable of generating mathematics knowledge artefacts; that students have access to this code appears assumed in the transcript, for in the next sentence the teacher adds "I think that's where a lot of people couldn't see this next jump". This suggests that *both* teacher and students are performing tasks which at this point draw on a knowledge base peripheral to the mathematics content knowledge bases of each. From the teacher's perspective, this knowledge base involves performing tasks relevant to both mathematics and teaching, 'mathematics teaching', and so, can be designated mathematics pedagogic content knowledge; from the student's perspective, the knowledge base in question also involves the performance of tasks relevant both to mathematics and teaching (for instance, knowing how to reconstitute a teaching artefact as a mathematical one, see above), and so also seems to be a kind of mathematics pedagogic content knowledge. Figure 3 summarises this analysis.

Figure 3
Proposed Model Indicating Relations among Teacher and Student Knowledge Bases



This episode also suggests that the teacher's ways of teaching, her functional knowledge claims within her pedagogic content knowledge base, are built around the dramatisation or staging of her mathematical knowledge base. For instance, in the lines 9-12 the teacher digresses to describe a set of events and imperatives which are drawn together in unity and lead to crisis, namely, "people couldn't see this next jump". This crisis could have been avoided, she states, had "people" only remembered to that when "dealing with complex numbers you need [sic] to put things together". This examples also shows that in her dramatisations of mathematical knowledge, mathematical operations are sequestered (Lave and Wenger, 1991) or bracketed within the roles of observer, critic, coach and commentator. Later in the episode, students are reminded that earlier in the lesson they were asked to "predict what argz_1z_2 *[writing 2.10]* is going to be" (lines 19-20). Then in lines 21-22 a conclusion is reached that the argument of the products is equal to the sum of the arguments. Next, in line 26, the teacher brings to the drama to a crisis with the question "So was our original hypothesis correct?" Students answer "no" and the teacher responds "No, because we predicted - you predicted or someone predicted - who do we blame? Steve?" (line 28). As with the previous example, the crisis around which this drama turns possesses an almost moral quality - something on which the concept of "blame" can be attached, even in jest. A second crisis ensues in lines 32-34. Here the transcript shows that the teacher uses a complex set of words, speech patterns, and hand, body and facial gestures to indicate that the outcome of this result is "dramatically different" (line 34) from "what you'd obviously think" (line 32). The teacher's choice of the second person pronoun is also interesting here, and perhaps gives a clue why she is choosing to be so exaggerated in her expressions at this point of the lesson. In her range of pronoun usages 'I' and 'we' are invariably used to indicate her, 'you' to indicate a student or students. Further it is almost always the case that a misunderstanding or misconception is attributed to 'you' in her discourse. This could mean that when she says "it's not what you'd obviously think [*teacher rubbing hands*]" (line 32), she is really saying "it's obvious that you'd fall into this trap". If so, then this provides an example of the teacher playfully teasing her students by setting up a contrasting relationship between an inner meaning and an outer meaning. This example of irony, echoes my previous observation, that mathematical knowledge is sequestered in the course of the lesson, it is bracketed by the need to foreground the dramatisation of mathematical knowledge (alias pedagogic content knowledge) and encourage 'in role' student behaviours. Such behaviours are seen when she says "Well, one person has made the next jump. Good. What do you think is the next stage? *[teacher talking to student]*"(lines 1-2), "No, because we predicted - you predicted or someone predicted - who do we blame? Steve?" (line 28). These analyses further illustrate Figure 3, by demonstrating the richness of the non-mathematical life of 'mathematics teaching' and indicating that both teacher and students share functional knowledge claims within the 'mathematics teaching' knowledge base. Mathematical knowledge is shown to stand in a peripheral relation to mathematics teaching.

This episode also provides information concerning the constructed nature of the shared teaching knowledge base. The scripted nature of her teacherly interactions becomes more clear. For instance: in lines 1-12 the teacher draws students into an active engagement with mathematical manipulations; in line 13 she prompts active reflection on the artefact these manipulations generate (*2.7*); then in lines 14-18 more manipulations follow, making use of an identity previously obtained; in lines 19-21 active reflection in which students are prompted to come to a view about the significance of their recent work for a task previously encountered, is again prompted; more related mathematical manipulations follow in lines 21-25; and finally, in lines 26-51, the teacher generates a (meta)narrative consisting of a commentary on the overarching mathematical strategies followed in the lesson up to this time, ideas that were used and tasks performed, and thereby reconstructs the context of the preceding work. This analysis indicates that her pattern of progress from manipulations to reflection is frequently repeated, and that students participate in these manoeuvres as the drama unfolds. This seems to point to the scripted nature of her performance and illustrates that the organisation of her exposition is purposeful and strategic, and that the logic of the script she follows allows her to produce, manipulate and regulate mathematics related knowledge artefacts for the purpose of teaching.

Note that the model presented in this figure indicates relationships among a broader range of knowledge bases than those discussed above (namely, those in shaded blocks). These refer, of course, to other components of teacher knowledge, for example, general pedagogic knowledge, curriculum knowledge, and so on (Shulman, 1986, 1987; Sternberg & Horvath, 1995; etc); and also student knowledge, attributions and beliefs relating to themselves as learners and mathematics as a body of knowledge, the various contexts of the school institution, their place within the social matrix of gender, culture and language values, and so on (Brown, 1997). Without referring to these matters in any detail, it will be recalled that the preceding analysis touched on the role of irony in building mathematics teaching knowledge claims. I would speculate that this, together with the good humour implicit in it, expresses the expectations and commitment of the teacher as she makes and implements her functional knowledge claims within the lesson.

CONCLUSIONS

In this paper I have attempted a detailed epistemological analysis of a lesson episode in order to see how mathematical knowledge is handled and how it stands in relation to teaching knowledge. Notwithstanding the obstacles faced (including unclear and uncertain terminology, conceptual confusion, methodological constraints - each of these suggesting areas for further study), evidence has been provided that mathematical knowledge stands in a peripheral relation to the lesson studied. The paper concludes by raising the question whether mathematics learning takes place because of this peripherality or despite it.

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